MOTION OF A SOLID CONTAINING A SPHERICAL DAMPER

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For the purpose of modeling the motion of a solid with a cavity filled with a viscous fluid, M. A. Lavrent'ev [1] has proposed a model in the form of a solid with a spherical cavity in which another solid, spherical in shape, is enclosed. The sphere is separated from the cavity walls by a small clearance in which viscous forces act (a lubricating film). This simple model with a finite number of degrees of freedom possesses certain mechanical properties of a solid with a cavity containing a viscous fluid. Study of this model is therefore of interest.

The present paper examines certain properties of the model, which will be termed a "solid with a damper". It is shown that for a high-viscosity lubricant the motion of a solid with a damper can be described by the same equations which pertain to the motion of a solid with a spherical cavity filled with a high-viscosity fluid. Expressions relating the parameters of the systems are obtained. If these relations are fulfilled, the systems become mechanically equivalent.

The steady motions of a free solid with a damper and their stability conditions are determined.
These motions and stability conditions hold for a body with a cavity filled with a viscous fluid [2].

§1. Let the solid G with the mass m_0 contain the spherical cavity D with the radius a. The cavity encloses a solid sphere with the mass m and a radius close to a. The mass distribution of the sphere posesses spherical symmetry (it is homogeneous, for example). The width h of the clearance between the sphere and the cavity walls is postulated small ($h \ll a$), so that the displacements of the center of the sphere relative to the center O_1 of the cavity D can be neglected (Fig. 1). We now derive the equations of motion of the system.

The equation of motion of the center of mass is $(m_0 + m)\mathbf{w} = \mathbf{F}$, where **w** is the acceleration of the system's center of mass, and **F** is the dominant vector of all external forces acting on the system.

Let O be any point rigidly coupled to the solid (for example, the system's center of mass, or a fixed point, if one exists). We introduce two systems of Cartesian coordinates: the $Oy_1y_2y_3$-system, whose axes move in arbitrarily prescribed manner (translational motion, for example), and the $Ox_1x_2x_3$-system,

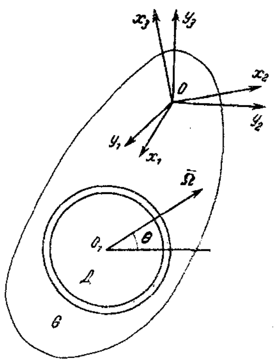

Fig. 1

which is rigidly coupled to the solid (Fig. 1). The moment equation in the $Oy_1y_2y_3$-system is taken as

$$\frac{d\mathbf{K}}{dt} = \mathbf{M}, \quad \mathbf{K} = \int\limits_{G+D} \mathbf{r} \times \mathbf{v}\, dm, \tag{1.1}$$

where t is time; K is the kinetic moment of the solid with a damper relative to the point O in the $Oy_1y_2y_3$-system; M is the principal moment relative to the point O of all external forces acting on the solid with a damper, in the same system of coordinates; r is a radius vector, read from the point O; v is the velocity in the $Oy_1y_2y_3$-coordinate system, dm is a mass element. The moment M comprises, in particular, the moment of inertial forces that is governed by the motion of the $Oy_1y_2y_3$-coordinate system. The velocitv v of any point of the system is $v = \omega x r + u$, where ω is the angular velocity of the solid relative to the Oy_1, y_2, y_3-system, and u is the velocity of this point relative to the $Ox_1x_2x_3$-system.

It is obvious that for points of thesolid $u = 0$, in which case formula (1.1) for K becomes

$$\mathbf{K} = \int\limits_{G+D} \mathbf{r} \times (\boldsymbol{\omega} \times \mathbf{r})\, dm + \mathbf{L}$$

$$= \mathbf{J} \cdot \boldsymbol{\omega} + \mathbf{L}, \quad \mathbf{L} = \int\limits_{D} \mathbf{r} \times \mathbf{u}\, dm. \tag{1.2}$$

Here J is the inertia tensor of the entire system relative to the point O, whose components are constant in the $Ox_1x_2x_3$-system. The point denotes the product of a tensor and a vector. The quantity **L**, termed a gyrostatic moment, represents the kinetic moment of the damper in the $Ox_1x_2x_3$-coordinate system. It can readily be seen that this moment does not depend on the choice of the pole, and is equal to

$$\mathbf{L} = I\boldsymbol{\Omega}, \quad \boldsymbol{\Omega} = \boldsymbol{\omega}_1 - \boldsymbol{\omega}. \tag{1.3}$$

where I is the moment of inertia of the damper relative to its diameter, and w and Ω are the angular velocities of the damper in the $Oy_1y_2y_3$- and $Ox_1x_2x_3$-systems, respectively.

We assume that the external force acting on the damper do not create a moment relative to its center. The moment M_1 of the forces of interaction between the damper and the solid relative to the point O_1 is postulated equal to $-k\Omega$, where k is a constant proportionality factor. Then the equation of motion of the damper with respect to its center becomes

$$I d\boldsymbol{\omega}_1 / dt = \mathbf{M}_1 = -k\boldsymbol{\Omega}, \quad k > 0. \tag{1.4}$$

We note that from the quantities appearing in Eq. (1.4) it is possible to construct the dimensionless criterion $R_1 = I(kT)$. This criterion is analogous to the Reynolds number for a solid with a cavity containing a fluid (see below); here, the constant T is the characteristic time of the process.

Equations (1.1–1.4) describe the motion of the solid with a damper in the $Oy_1y_2y_3$-coordinate system. Generally, they should be supplemented by the usual kinematic relations, which we will not write out. We rewrite Eqs. (1.1–1.4), denoting by primes the derivative in the $Ox_1x_2x_3$-coordinate system:

$$\mathbf{K}' + \boldsymbol{\omega} \times \mathbf{K} = \mathbf{M}, \quad \mathbf{K} = \mathbf{J}\cdot\boldsymbol{\omega} + I(\boldsymbol{\omega}_1 - \boldsymbol{\omega}),$$

$$I\boldsymbol{\omega}_1' + I(\boldsymbol{\omega} \times \boldsymbol{\omega}_1) = k(\boldsymbol{\omega} - \boldsymbol{\omega}_1). \qquad (1.5)$$

We calculate k, assuming that the interaction between the damper and the solid occurs through a spherical film of an incompressible viscous fluid with the density ρ_1 and the kinematic viscosity ν_1. In the $Ox_1x_2x_3$-coordinate system a point, on the damper diameter that forms the angle θ with the vector Ω (Fig. 1), has the velocity $\Omega a \sin\theta$. This is accompanied by the development in the fluidfilm (with the thickness h) of the velocity gradient $\Omega a \sin\theta$ that gives rise to tangential stress at the damper surface, equal to $\rho_1\nu_1\Omega a \sin\theta/h$. We calculate the magnitude of the moment created by these stresses with respect to the point O_1, and having compared it with formula (1.4), we determine k:

$$\mathbf{M}_1 = \int_0^\pi \frac{\rho_1\nu_1\Omega a \sin\theta}{h} 2\pi a^3 \sin^2\theta \, d\theta = \frac{8\pi\rho_1\nu_1 a^4}{3h}\Omega,$$

$$k = (8\pi/3)\rho_1\nu_1 a^4 h^{-1}. \qquad (1.6)$$

§ 2. Let the angular velocities ω and ω_1 be on the order of T^{-1} (where T is the characteristic time of the process), let their derivative ω' and ω_1 in the $Ox_1x_2x_3$-coordinate system on the order of T^{-2}, and let ω'' and ω''_1 be on the order of T^{-3}, Furthermore, we assume that R_1 is small ($R_1 \ll 1$). Without loss of generality, we can take T as the unit of time, the damper radius a as the unit length, and I/a as the unit of mass. Then the moment of inertia I, the angular velocities ω and ω_1, and their first and second derivatives are values on the order of unity, while $R_1 = 1/k$, where $k \gg 1$ (a high-viscosity lubricant) We rewrite Eq. (1.4), and differentiate both its sides with respect to time in the Ox_1x $Ox_1x_2x_3$- coordinate

$$I\boldsymbol{\omega}_1' + I\boldsymbol{\omega} \times \boldsymbol{\omega}_1 = -k\Omega,$$

$$I\boldsymbol{\omega}_1'' + I\boldsymbol{\omega}' \times \boldsymbol{\omega}_1 + I\boldsymbol{\omega} \times \boldsymbol{\omega}_1' = -k\Omega'. \qquad (2.1)$$

According to our estimates, the left-hand sides of (2.1) are quantities on the order of unity, and therefore $|\Omega| \sim |\Omega'| \sim k^{-1} \ll 1$. We now substitute $\omega_1 = \omega + \Omega$ into the first equation of (2.1) and extress Ω with accuracy to the smallest highest-order infinitesimals

$$\Omega = -Ik^{-1}\omega' + O(k^{-2}) \qquad (k \gg 1).$$

Substituting this formula into equality (1.3), we obtain

$$\mathbf{L} = -I^2 k^{-1}(d\boldsymbol{\omega}/dt) + O(R_1^2) \qquad (R_1 \ll 1)$$

which takes into account that $dw/dt = \omega'$.

We examine now the motion of a solid with a cavity completely filled with an incompressible viscous fluid with the density ρ and the kinematic viscosity ν. Then, Eqs. (1.1) and (1.2) with all their introduced notation continue to hold; however, L no longer is defined by Eq. (1.3).

Let us assume as before that ω', ω, ω'' are on the order of T^{-1}, T^{-2}, T^{-3}, respectively, where T is the characteristic time of the process. Furthermore, let the Reynolds number be small ($R = l^2/(\nu T) \ll 1$ (l is the characteristic dimension of the cavity), and let all the external forces acting on the fluid in the $Oy_1y_2y_3$-coordinate system be potential forces. Then, as shown in [3], the equality

$$\mathbf{L} = -\rho\nu^{-1}\mathbf{P}(d\boldsymbol{\omega}/dt) + O(R^2) \qquad (R \ll 1). \qquad (2.3)$$

will hold.

Here, **P** is a constant tensor that depends on the configuration the cavity and that characterizes energy dissipation due to viscosity. General expressions for the components of the tensor P and some of its properties are given in [3], together with specific formulas for a number of cavity configurations. According to [3], for a spherical cavity of radius a we have

$$\mathbf{P} = P\mathbf{E}, \quad P = \frac{8\pi a^7}{525}, \quad \mathbf{L} = -\frac{\rho}{\nu} P \frac{d\boldsymbol{\omega}}{dt} + O(R^2),$$

$$R = \frac{a^2}{\nu T} \ll 1, \qquad (2.4)$$

where E is a unit tensor.

Formula (2.4) is completely analogous to (2.2). However, equality (2.2) or (2.4), in combination with Eqs. (1.1), as well as with the kinematic relations, describes completely the motion of a solid in the $Oy_1y_2y_3$-coordinate system.

Hence, for our assumptions, the motion of a solid with a damper and that of a solid with a spherical cavity containing a liquid are described by the same equations.

To achieve complete mechanical equivalence of the systems (for the same solid, the same cavity radius a, and the same external forces and moments F and M), it is required that: 1) the mass of the damper be equal to that of the fluid in the cavity (to ensure equivalence of the equations of motion of the center of mass), 2) the moments of inertia with respect to the diameter of the damper and the liquid be alike (to ensure equality of the inertia tensor J of the entire system in Eq. (1.2)), and 3) the coefficients in front of $d\omega/dt$ in Eqs. (2.2) and (2.4) be equal. The mass and the moment of inertia of the lubrication film are neglected. On this basis, we obtain the following equalities

$$m = \frac{4}{3}\pi\rho a^3, \quad I = \frac{8}{15}\pi\rho a^5, \quad k = \frac{\nu I^2}{\rho P} = \frac{56}{3}\pi\rho a^3. \qquad (2.5)$$

which (for our assumptions) are necessary and sufficient for equivalence of the systems.

With formula (1.6) the last equality in (2.5) can be transformed into

$$\rho_1\nu_1(a/h) = 7\rho\nu,$$

This formula, just like (1.6), is valid for $h \ll a$. We note that when the equalities in (2.5) are fulfilled we have the following relation between the Reynolds numbers

$$R = 35R_1, \quad R_1 = I/(kT), \quad R = a^2/(\nu T).$$

If, in modeling, we drop the condition that the solids identical, a solid with a damper at $R \ll 1$ can model a solid with a viscous fluid at $R \ll 1$ for those cavity configurations for which the tensor P has the form $P = PE$, where p is a scalar (e.g., for a sphere, a cube, etc.). If, on the other hand, a solid with a cavity filled with a viscous fluid at $R \ll 1$ a viscous fluid performs a plane motion. such as rotating about the fixed x_3 axis then for any cavity configuration it can be modeled by a solid with a damper, in which case the damper can be any axisymmetric solid with an axis of symmetry parallel to x_3.

Lastly, a solid with a cavity of arbitrary configuration (with any tensor P) filled with a viscous fluids at $R \ll 1$ can be modeled by means of a solid containing three or more axisymmetric dampers (flywheels with viscous damping). Let the solid containg three flywheels whose axes are mutually perpendicular and parallel to the major axes of the tensor P. Then it is easy to show that for the solid with the fluid (at $R \ll 1$ to be equivalent to the solid with the flywheels, it is sufficient (for like external forces and moments, as well as for the same masses and inertia tensors in both systems) that conditions be fulfilled analogous to (2.5), namely, $\rho\nu^{-1}P_{jj} = I^2{}_j/k_j$ for $j = 1, 2, 3$. Here, P_{jj} are the principal values of the tensor P; I_j is the moment of inertia of the j-th flywheel, with respect to its axis k_j is the damping factor of the j-th flywheel, i. e., the proportionality factor between the moment of the forces of interaction of the flywheel with the solid and its angular velocity with the respect to the solid.

In [3] it was shown that a $R \ll 1$, Eqs. (1.1), (1.2), and (2.3) can be simplified, In the same paper, certain motions of a solid with a cavity filled with a viscous fluid were studied in the case of $R \ll 1$. These considerations are fully applicable to a solid with a damper at $R_1 \ll 1$. From the results of [3], in particular, it follows that rotation about the axis of the maximum moment of inertia of the entire system is the only stable steady rotation of a free solid with a damper at $R_1 \ll 1$.

§ 3. Let us examine the motion of a free solid with a damper, without limiting ourselves to the condition $R_1 \ll 1$. Postulating $M = 0$ in Eqs. (1.5), and subtracting the third equation from the first, we obtain

$$\mathbf{J}_0 \cdot \boldsymbol{\omega}' + \boldsymbol{\omega} \times (\mathbf{J}_0 \cdot \boldsymbol{\omega}) = k(\boldsymbol{\omega}_1 - \boldsymbol{\omega}),$$
$$I\boldsymbol{\omega}_1' + I\boldsymbol{\omega} \times \boldsymbol{\omega}_1 = k(\boldsymbol{\omega} - \boldsymbol{\omega}_1), \quad (\mathbf{J}_0 = \mathbf{J} - I\mathbf{E}). \qquad (3.1)$$

Here, J_0 is the inertia tensor of the system with respect to the point O, provided the entire mass of the damper is concentrated at its center. Equations (3.1) form a closed system. They can describe the motion of a free solid about a fixed point (if O is a fixed point) or about the center of mass, if O is the center of mass of the system, and the $Oy_1y_2y_3$-coordinate system moves translationally.

Let us couple the $Ox_1x_2x_3$-coordinate system to the major axes of the inertia tensor J of the system with respect to the point O. These axes obviously will also be the major axes fo the tensor $\mathbf{J}_0$. Let p, q, and r denote the projections of the vector ω onto the x_1x_2 and x_3 axes respectively, let p_1, q_1, and r_1 denote the projections of the vector w_1 onto the same axes, let A, B, and C denote the principal moments of inertia of the entire system with respect to these axes, and let A_0, B_0, and C_0 denote the principal values of the tensor J_0 in the same axes (values equal to $A - I$, $B - I$, and $C - I$, respectively). In the scalar notation, Eqs. (3.1) will take the form

$$A_0p' + (C_0 - B_0)\, qr = k(p_1 - p),$$
$$I(p'_1 + qr_1 - rq_1) = k(p - p_1),$$
$$B_0q' + (A_0 - C_0)\, rp = k(q_1 - q),$$
$$I(q'_1 + rp_1 - pr_1) = k(q - q_1),$$
$$C_0r' + (B_0 - A_0)\, pq = k(r_1 - r),$$
$$I(r'_1 + pq_1 - qp_1) = k(r - r_1). \qquad (3.2)$$

Let us determine the possible steady motions of the solid. If ω is a constant $\omega' = 0$), from the first equation in (3.1) it follows that ω_1 is also a constant, and $\omega'_1 = 0$. Then, by scalar multiplication of both sides of the second equation in (3.1) by $\omega_1 - \omega$, we obtain $\omega_1 - \omega$. From Eqs. (3.2) it can be seen that such motion is possible only for the case in which rotation occurs about one of the system's major axes of inertia. Thus, the only possible steady motions of both the system and the solid with a viscous fluid are uniform rotations of the system as a solid entity about one of the major axes of inerita.

We shall examine the stability of these motions. Let the uperturbed motion (rotation of the system about the x_1-axis at the constant angular velocity ω_0) be described by the equalities

$$p = p_1 = \omega_0, \quad q = q_1 = r = r_1 = 0$$
$$(\omega_0 \neq 0). \qquad (3.3)$$

We set $p = \omega_0 + x$ and $p_1 = \omega_0 + y$ in the perturbed motion, and linearize, Eq. (3.2) about the solution of (3.3)

$$A_0x' = k(y - x),\ Iy' = k(x - y),$$
$$B_0q' + (A_0 - C_0)\,\omega_0 r = k(q_1 - q),$$
$$C_0r' + (B_0 - A_0)\,\omega_0 q = k(r_1 - r),$$
$$Iq'_1 + I\omega_0(r - r_1) = k(q - q_1),$$
$$Ir'_1 + I\omega_0(q_1 - q) = k(r_1 - r). \qquad (3.4)$$

The first two equation in (3.4) are independent of the remaining four equations, so that the characteristic equation of system (3.4) breaks down into two equations. After expansion of the determinants, the characteristic equations reduce to the form

$$A_0I\lambda^2 + k(A_0 + I)\lambda = 0,$$
$$a_0\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0,$$
$$a_0 = B_0C_0I^2,\ a_1 = Ik(2B_0C_0 + B_0I + C_0I)$$
$$a_2 = I^2\omega_0^2\,[(A_0 - B_0)(A_0 - C_0) +$$
$$+ B_0C_0] + k^2(B_0 + I)(C_0 + I),$$
$$a_3 = I\omega_0^2k\,[2(A_0 - B_0)(A_0 - C_0) +$$

$$+ I(A_0 - B_0) + I(A_0 - C_0)],$$

$$a_4 = \omega_0^2 (I^2\omega_0^2 + k^2)(A_0 - B_0)(A_0 - C_0) \qquad (3.5)$$

The meaning of the zero root of the first equation in (3.5) is that an initial disturbance of the kinetic moment of the entire system is retained constant. Because of the presence of the zero root, analysis of the linerarized system (3.4) yields necessary but not sufficient conditions for stability of motion (3.3). For the motion to be stable, the real parts of all the roots λ of Eqs. (3.5) must be nonpositive: Re $\lambda \le 0$. For the roots of the first equation in (3.5) this condition is fulfilled when $k \ge 0$. For it also to hold for the second equation, the Lienard-Chipard conditions [4] must be fulfilled: Here equality signs are permissible, since we require the inequality Re $\lambda \le 0$ and not Re $\lambda > 0$, as is usually the case.

Inasmuch as $a_0 > 0$, the Lienard-Chipard conditions for the second equation in (3.5) will have the form [4]

$$a_1 \geqslant 0, \ a_2 \geqslant 0, \ a_4 \geqslant 0,$$

$$a_1 a_2 a_3 \geqslant a_0 a_3^2 + a_1^2 a_4. \qquad (3.6)$$

With use of (3.5), the last inequality in (3.6) can be reduced after cumbersome but elementary algebraic transformations to the form

$$2I^2\omega_0^2 (A_0 - B_0 - C_0)^2 [B_0 (A_0 - C_0) + C_0(A_0 - B_0)] +$$
$$+ k^2 (2B_0C_0 + B_0 I + C_0 I) [(A_0 - B_0)(B_0 + I) + (A_0 - C_0)(C_0 + I)] \geqslant 0. \qquad (3.7)$$

From condition $a_4 > 0$, it follows that one must have either $A_0 \le B_0 \le$ and $A_0 \le C_0$ or $A_0 \ge B_0$ and $A_0 \ge C_0$. However, we readily see that in the first case condition (3.7) no longer holds, while in the second case none of the conditions (3.6) and (3.7) are fulfilled for $k \ge 0$. The inequalitities $k \ge 0$, $A_0 \ge B_0$, and $A_0, \ge C_0$ are precisely the necessary stability conditions for motion (3.3).

Concerning the sufficient conditions, it can be said that system (3.2) has the first integral

$$K^2 = (J_0\omega + I\omega_1)^2 = (A_0 p + I p_1)^2 + (B_0 q + I q_1)^2 + + (C_0 r + I r_1)^2 = \text{const}, \qquad (3.8)$$

which expresses conservation of the kinetic moment of the entire system. We can readily see that by virtue of Eqs. (3.2) the kinetic energy E, defined by the equality

$$2E = A_0 p^2 + B_0 q^2 + C_0 r^2 + I(p_1^2 + q_1^2 + r_1^2). \quad (3.9)$$

does not increase during motion—i.e., $E' \le 0$ when $k \ge 0$. Using the idea of Chetaev's method, we construct the Lyapunov function

$$V = 2(A_0 + I) E - K^2 + [K^2 - (A_0 + I)^2 \omega_0^2]^2. \quad (3.10)$$

It is not difficult to see that the function V vanishes for unperturbed motion (3.3). For perturbed motion, as above, we substitute $p = \omega_0 + x$ and $p_1 = \omega_0 + y$ into Eqs. (3.8) and (3.9), and then write V in (3.10) as a function of the variables x, y, q, q_1, r, and r_1. With this, the linear terms cancel each other out, and after collecting like terms, we obtain

$$V = [A_0 I (x - y)^2 + 4\omega_0^2 (A_0 + I)^2 (A_0 x + I y)^2] +$$
$$+ [B_0 (A_0 + I - B) q^2 - 2B_0 I q q_1 + A_0 I q_1^2] +$$
$$+ [C_0 (A_0 + I - C_0) r^2 - 2C_0 I r r_1 + A_0 I r_1^2] + \ldots \quad (3.11)$$

The points denote terms of the third and higher orders of smallness. The first square brackets in (3.11) are a positive definite (for $\omega_0 \ne 0$) quadratic form from x and y. For the other two quadratic forms in (3.11) to be positive definite, it is enough to require that

$$A_0 (A_0 + I - B_0) > B_0 I, \quad A_0 (A_0 + I - C_0) > C_0 I$$

Removing the parentheses and canceling by the factor $A_0 + I$, these inequalities can be reduced to the form $A_0 > B_0$ and $A_0 > C_0$. For these conditions the function V will be positive definite. Because $(K^2)' = 0$, while $E' \le 0$, the derivative of function (3.10) is nonpositive by virtue of the equations of motion: $V' \le 0$ for $k \ge 0$. According to Lyapunov's theorem, motion will be stable for the above conditions $w_0 \ne 0, k \ge 0$, $A_0 > C_0$. We note that the inequalities $A_0 > B_0$ and $A_0 > C_0$ are equilvalent to the inequalities $A \ge B$ and $A > C$ for the principal moments of inertia of the entire system.

Thus, for the steady rotation of a free solid with a damper about x_1-axis (motion (3.3)) to be stable at $\omega_0 \ne 0$ and $k \ge 0$, the conditions $A \ge B$, $A \ge C$ must be fulfilled, and it is sufficient for the rigorous inequalities $A > B$ and $A > C$ to occur. In other words, stable steady rotation of a free solid with a damper is possible only about the axis of the maximum principal moment of inertia. We note that in the general case the same sufficient stability condition [2] ($A > B$, $A > C$) occur for the steady rotations of a free solid with a cavity filled with liquid, while for a high viscosity fluid [3] the necessary stability conditions ($A \ge B$ and $A \ge C$) are required in addition to the sufficient conditions.

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